

# Energy Efficient Building Climate Control using Stochastic Model Predictive Control and Weather Predictions

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**Abstract**—One of the most critical challenges facing society today is climate change and thus the need to realize massive energy savings. Since buildings account for about 40% of global final energy use, energy efficient building climate control can have an important contribution. In this paper we develop and analyze a Stochastic Model Predictive Control (SMPC) strategy for building climate control that takes into account weather predictions to increase energy efficiency while respecting constraints resulting from desired occupant comfort. We investigate a bilinear model under stochastic uncertainty with probabilistic, time varying constraints. We report on the assessment of this control strategy in a large-scale simulation study where the control performance with different building variants and under different weather conditions is studied. For selected cases the SMPC approach is analyzed in detail and shown to significantly outperform current control practice.

## I. INTRODUCTION

### A. Integrated Room Automation

In building climate control heating, ventilation, and air-conditioning (HVAC) systems are employed to keep room temperature within a predefined range, the so-called comfort range. In this paper we focus on *Integrated Room Automation*, where the building system consists of an HVAC-system, an automated lighting system, and a blind positioning system [4], [9]. The control task is to keep the room temperature as well as  $CO_2$  and illuminance levels within a predefined comfort range, which can be fulfilled with a set of different actuators. The actuators differ in terms of response time and effectiveness, in their dependence on weather conditions (e.g. cooling tower or blinds), and in energy costs. The goal is to optimally choose the actuator settings depending on future weather conditions in order to fulfill the comfort requirements and minimize energy costs.

### B. Assessment of Control Strategies

Aiming at investigating how much energy can be saved with advanced control techniques we compare Model Predictive Control (MPC) strategies taking into account weather predictions with current best-practice control. For this assessment we use BACLab, a MATLAB-based modeling

and simulation environment for building climate control developed within the OptiControl<sup>1</sup> project, which focuses on the development of predictive control strategies for building climate control. A bilinear model is used for both simulation and control. The crucial part of the control problem is how to deal with the inherent uncertainty due to weather predictions. The following controllers are assessed:

- *Rule Based Control* (RBC): Current practice. The control inputs are defined with simple rules: “if *condition* then *action*”.
- MPC: Two different MPC schemes are considered. The first strategy follows common practice, which is to simply neglect the uncertainty in the problem and is therefore termed *Certainty Equivalence* (CE). The second strategy takes into account the uncertainty in the controller directly and solves a stochastic MPC (SMPC) problem. For this, we follow the approach introduced in [11].
- *Performance Bound* (PB): Optimal control action given perfect knowledge of future weather. This is an ultimate bound on the performance of any controller, and thus used as a theoretical benchmark.

### C. Outline

In Section II the modeling is described in detail. This is divided into two parts, the building modeling and the weather uncertainty modeling. In Section III the control strategies are presented. First, the RBC strategy is explained, then the MPC problem is posed. This can be solved by neglecting the uncertainty as in CE or by directly taking it into account as in SMPC. Both approaches are presented in detail. Finally, the PB is introduced. Section IV introduces the concept of the controller assessment and describes the setup of the large-scale simulation study. The simulation results are presented in Section V.

### D. Notation

The real number set is denoted by  $\mathbb{R}$ , the set of non-negative integers by  $\mathbb{N}$  ( $\mathbb{N}_+ := \mathbb{N} \setminus \{0\}$ ). For matrices  $A$  and  $B$  of equal dimension inequalities  $A\{<, \leq, >, \geq\}B$  hold component-wise. The expectation of a stochastic variable  $w$  given the observation  $\tau$  is denoted by  $E[w|\tau]$ . The probability of an event  $\rho$  is denoted by  $P[\rho]$ .

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## II. MODELING

### A. Building Model

For computing the building-wide energy use it is common practice to sum the energy uses of single rooms or building zones [4]. We follow this approach and focus on the dynamics of a single room. We first explain the building thermal dynamics in detail and then the different actuators.

**Remark 1:** Illuminance and CO<sub>2</sub> concentration were modeled by instantaneous responses since the time constants involved were much smaller than the hourly time step employed for our modeling and simulations and modelling details of these are omitted for brevity. The interested reader can find the details on this in [10].

The principle of the thermal dynamics modeling can easily be described with a small example as given in Figure 1. The room can be thought of as network of first-order systems, where the nodes are the states  $x$  and these are representing the room temperature or the temperatures in the walls, floor or ceiling. Then the heat transfer rate is given by

$$\begin{aligned} \frac{dQ}{dt} &= K_{ie} \cdot (\vartheta_e - \vartheta_i) \\ \Rightarrow \underbrace{\frac{dQ}{d\vartheta_i}}_{C_i} \cdot \frac{d\vartheta_i}{dt} &= K_{ie} \cdot (\vartheta_e - \vartheta_i), \end{aligned} \quad (1)$$

where  $t$  denotes the time,  $\vartheta_i$  and  $\vartheta_e$  are the temperatures in layers  $i$  and  $e$  respectively,  $Q$  is thermal energy, and  $C_i$  denotes the thermal capacitance of layer  $i$ . The total heat transmission coefficient  $K_{ie}$  is computed as

$$\frac{1}{K_{ie}} = \frac{1}{K_i} + \frac{1}{K_e}, \quad (2)$$

where the heat transmission coefficients  $K_i$  and  $K_e$  depend on the materials of  $i$  and  $e$  as well as on the cross sectional area of the heat transmission. For each node, i.e. state, such a differential equation as in (1) is formulated. Control actions were introduced by assuming that selected resistances were variable. For example, solar heat gains and luminous flux through the windows were assumed to vary linearly with blinds position, i.e. the corresponding resistances were multiplied with an input  $u \in [0, 1]$ . This leads to a bilinear model, i.e. bilinear in state and input as well as in disturbance and input. A detailed description of the building model can be found in [10].

Concerning the actuators, we investigated several variants of building systems in integrated room automation. All system variants had the basic actuators blind positioning and electric lighting. They employed different combinations of the following subsystems:

- Heating: radiators / mechanical ventilation / floor heating / TABS<sup>2</sup>

<sup>2</sup>TABS = Thermally activated building system, i.e. the building mass is incorporated as thermal storage for heating and cooling purposes and activated by a tube-system located in the slabs.

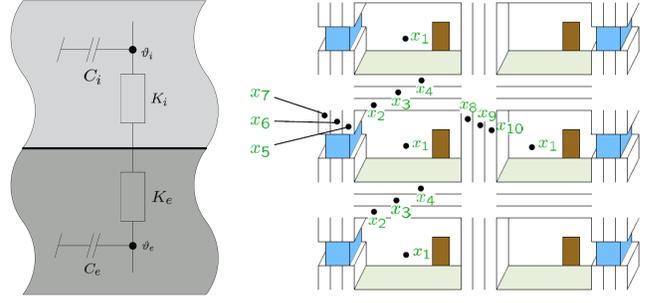


Fig. 1. **Heat transmission between nodes.** The modeling is based on the description of heat transmission between nodes (left) that are representing the temperatures at different locations in the building (right).

- Cooling: evaporative cooling (wet cooling tower) / mechanical ventilation / chilled ceiling / TABS
- Ventilation: with/without mechanical ventilation (including energy recovery); with/without natural night-time ventilation

The delivered heating or cooling power, the used air change rates as well as lighting and blind positioning correspond to the control inputs  $u$ . The control task consists of finding the optimal combination of inputs that differed in their weather dependence, dynamical effects and energy use. For example mechanical ventilation, which provides the room with fresh air to guarantee indoor air quality, can be used both for cooling and for heating, depending on the weather conditions. But heating can also be done with radiators, which are independent of weather conditions. TABS can be used for heating and cooling but are much slower compared to ventilation or radiators etc. Further details on the experimental setup can be found in [9].

**Assumption 1:** The room dynamics are described as

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k + \dots \\ &\dots + B_v v_k + \sum_{i=1}^m [(B_{vu,i} v_k + B_{xu,i} x_k) u_{k,i}] \end{aligned} \quad (3)$$

where  $x_k \in \mathbb{R}^n$  is the state,  $u_k \in \mathbb{R}^m$  is the input, and  $v_k \in \mathbb{R}^p$  is the weather input at time step  $k$ , and the matrices  $A, B, B_v, B_{vu,i}$ , and  $B_{xu,i}$  are of appropriate sizes. The sampling time is 1 hour.

The overall building model was validated by building experts [10] and its dynamic response compared to simulations with TRNSYS<sup>3</sup>, which is a well known simulation software for buildings and HVAC systems. It was found that the model captures sufficiently well the relevant behavior of a building [10].

### B. Weather Uncertainty Model

The weather predictions were given by archived forecasts of the COSMO-7 numerical weather prediction model operated by MeteoSwiss. The data comprised the outside air temperature, the wetbulb temperature and the incoming solar

<sup>3</sup><http://sel.me.wisc.edu/trnsys/>

radiation. COSMO-7 delivers hourly predictions for the next three days with an update cycle of 12 hours [6]. The major challenge from a control point of view with using numerical weather predictions lies in their inherent uncertainty due to the stochastic nature of atmospheric processes, the imperfect knowledge of the weather models initial conditions as well as modeling errors. The actual disturbance acting on the building can be decomposed as

$$v_k = \bar{v}_k + \tilde{v}_k, \quad (4)$$

where  $\bar{v}_k$  is the COSMO-7 weather prediction and  $\tilde{v}_k$  is the prediction error at each time step  $k$ . In order to improve the estimation of future disturbances acting on the building the following autoregressive model driven by Gaussian noise was identified based on the archived weather predictions and corresponding in-situ measurements

$$\tilde{v}_{k+1} = F\tilde{v}_k + w_k, \quad (5)$$

where  $F \in \mathbb{R}^{p \times p}$  and  $w_k \in \mathbb{R}^p$ .

**Assumption 2:** The disturbance  $w_k$  follows a Gaussian distribution,  $w_k \sim \mathcal{N}(\bar{w}_k, \Sigma \Sigma^T)$ ,  $\forall k$ .

Testing the randomness of residuals showed that the goodness of fit was satisfying for all investigated cases, i.e. autocorrelation coefficients for the the residuals did not differ significantly from zero.

The model in (5) is used twofold: first, for augmentation of the controller model such that it accounted for the effects of the uncertain weather predictions on the building's dynamics, and second for continuous correction of the COSMO-7 weather predictions based on hourly weather measurements with a standard Kalman filter.

### C. Overall Model

The dynamic behavior of the building is nonlinear; in this case bilinear between inputs, states and weather parameters. Non-linearities in the dynamic equations of an MPC problem will generally result in a non-convex optimization, which can be extremely difficult to solve. The approach that we take here is a form of Sequential Linear Programming (SLP) for solving nonlinear problems in which we iteratively linearize the non-convex constraints around the current solution, solve the optimization problem and repeat until a convergence condition is met [3]. To keep formulations simple, we will assume for the remainder of the paper that we do the linearization at each hourly time step  $k$ , which results in the new input matrix  $B_{u,k}$  and formulate the problem for the linear system of the form

$$x_{k+1} = Ax_k + B_u u_k + B_v v_k. \quad (6)$$

## III. CONTROL STRATEGIES

In this section we present the different control strategies that are considered in our assessment. These are RBC, which is current practice, MPC, which takes into account weather predictions, and PB, which is a theoretical benchmark.

### A. Rule Based Control

The standard strategy in current practice and used by, amongst others, Siemens Building Technologies is rule-based control [5]. As the name indicates, RBC determines all control inputs based on a series of rules of the kind “if *condition* then *action*”. As a benchmark we used here RBC-5 as defined in [7]. This is the currently best RBC controller known to us that assumes hourly blind movement as the other control strategies considered in this study.

### B. Model Predictive Control Approach

For the MPC approach, we employ the model of (6).

**Remark 2:** We substitute (4) in (6) and use (5) to extend the model, such that the resulting model depends linearly on the Gaussian disturbance  $w$ .

Consider the prediction horizon  $N \in \mathbb{N}_+$  and define

$$\begin{aligned} \mathbf{x} &:= [x_0^T, \dots, x_N^T]^T \in \mathbb{R}^{(N+1)n} \\ \mathbf{u} &:= [u_0^T, \dots, u_{N-1}^T]^T \in \mathbb{R}^{Nm} \\ \mathbf{w} &:= [w_0^T, \dots, w_{N-1}^T]^T \in \mathbb{R}^{Np} \\ \bar{\mathbf{w}} &:= [\bar{w}_0^T, \dots, \bar{w}_{N-1}^T]^T \in \mathbb{R}^{Np} \end{aligned}$$

and prediction dynamics matrices  $\mathbf{A}$ ,  $\mathbf{B}$  and  $\mathbf{E}$  such that

$$\mathbf{x} = \mathbf{A}x_0 + \mathbf{B}\mathbf{u} + \mathbf{E}\mathbf{w}.$$

We assume that all inputs  $u$  can vary between zero and an upper bound (full operation), which defines hard constraints on the inputs. The room temperature is constrained to lie within upper and lower bounds. Motivated by European building standards, e.g. [14], we do not require constraints to be satisfied at all times, but only with a predefined probability, which is formulated with so-called chance constraints.

**Assumption 3:** The constraints on inputs  $\mathbf{u}$  and states  $\mathbf{x}$  over the prediction horizon  $N$  are

$$\begin{aligned} \mathbf{S}\mathbf{u} &\leq \mathbf{s} \\ \wedge \quad \mathbf{G}\mathbf{x} &\leq \mathbf{g}, \end{aligned} \quad (7)$$

where  $\mathbf{S} \in \mathbb{R}^{q \times mN}$ ,  $\mathbf{s} \in \mathbb{R}^q$ ,  $\mathbf{G} \in \mathbb{R}^{r \times n(N+1)}$ , and  $\mathbf{g} \in \mathbb{R}^r$  and the state constraints are applied according to the definition in the standards as chance constraints

$$P[\mathbf{G}_i \mathbf{x} \leq \mathbf{g}_i] \geq 1 - \alpha_i, \quad \forall i = 1, \dots, r, \quad (8)$$

where  $1 - \alpha_i$  with  $\alpha_i \in [0, 1]$  denotes the probability level.

This means that we formulate the chance constraints on the states as individual chance constraints, i.e. each row  $i$  has to be individually fulfilled with the probability  $1 - \alpha_i$ .

For some initial state  $x_0$  the control objective is to minimize energy usage.

**Assumption 4:** A linear cost function  $V : \mathbb{R} \rightarrow \mathbb{R}$

$$V(x_0, u_0, \dots, u_{N-1}) := \sum_{k=0}^{N-1} c^T \cdot u_k \quad (9)$$

is assumed, where  $x_0 \in \mathbb{R}^n$  is the initial state and  $c \in \mathbb{R}^m$  is the cost of the different actuators, i.e. a scaling factor, that

considers the non-renewable primary energy usage of each actuator, see [9].

The optimal control input  $u$  over the prediction horizon  $N$  is determined by solving MPC Problem 1.

**Problem 1:**

$$\begin{aligned}
u^*(x_0)_{MPC} &:= \arg \min_{\mathbf{u}} E[\mathbf{c}^T \mathbf{u} | x_0] \\
s.t. \quad &P[\mathbf{G}_i(\mathbf{A}x_0 + \mathbf{B}\mathbf{u} + \mathbf{E}\mathbf{w}) \leq \mathbf{g}_i] \geq 1 - \alpha_i \quad (10) \\
&\forall i = 1, \dots, r \\
&\mathbf{S}\mathbf{u} \leq \mathbf{s}
\end{aligned}$$

where  $\mathbf{c} \in \mathbb{R}^{Nm}$  denotes the cost vector for the whole horizon. Problem 1 is a dynamic programming problem, it is however not clear how to solve it since it depends on the disturbance  $\mathbf{w}$  which follows a Gaussian distribution. There are two principal ways to proceed: 1) The standard procedure is to assume  $\mathbf{w} = \bar{\mathbf{w}}$  and solve a deterministic MPC problem. This problem is known as Certainty Equivalence. 2) We follow the approach introduced in [11] and solve the stochastic MPC problem approximately.

1) *Certainty Equivalence:* With the assumption that  $\mathbf{w} = \bar{\mathbf{w}}$ , Problem 1 simplifies to

**Problem 2:**

$$\begin{aligned}
u^*(x_0)_{CE} &:= \arg \min_{\mathbf{u}} \mathbf{c}^T \mathbf{u} \\
s.t. \quad &\mathbf{G}(\mathbf{A}x_0 + \mathbf{B}\mathbf{u} + \mathbf{E}\bar{\mathbf{w}}) \leq \mathbf{g} \quad (11) \\
&\mathbf{S}\mathbf{u} \leq \mathbf{s}
\end{aligned}$$

This is now a deterministic problem known as nominal MPC problem and is readily solvable.

**Remark 3:** Note that the state constraints are only guaranteed to be satisfied for  $\mathbf{w} = \bar{\mathbf{w}}$ . Thus, other outcomes of  $\mathbf{w}$  are likely to violate the constraints.

2) *Stochastic Model Predictive Control:* Following the assumption that the disturbance  $\mathbf{w}$  is normally distributed, we get a stochastic MPC problem which is not readily solvable. We reformulate and approximate the stochastic control problem in two steps. First, we approximate the dynamic programming problem and second, we define a convex, deterministic reformulation of the probabilistic constraints in order to cast the SMPC Problem 1 as a convex and tractable optimization problem, which can be solved at each time step. For approximating the dynamic programming problem we employ affine disturbance feedback, which has shown good performance in robust MPC problems [1], [2].

With affine disturbance feedback, the control inputs are parameterized as affine functions of the disturbance sequence as follows

$$\begin{aligned}
u_i &= h_i + \sum_{j=0}^{i-1} M_{i,j} w_j \\
h_i &\in \mathbb{R}^m \\
M_{i,j} &\in \mathbb{R}^{m \times p} \quad \forall j \in \mathbb{N}_0^k
\end{aligned}$$

$\forall (i, j) \in \mathbb{N} \times \mathbb{N}_0^{N-1}$ . By doing so, the optimization in MPC Problem 1 can be solved in a computationally efficient fashion using convex optimization methods.

In matrix form this leads to

$$\begin{aligned}
\mathbf{u} &= \mathbf{M}\mathbf{w} + \mathbf{h} \quad (12) \\
\mathbf{M} &:= \begin{bmatrix} 0 & \dots & \dots & 0 \\ M_{1,0} & 0 & \dots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ M_{N-1,0} & \dots & M_{N-1,N-2} & 0 \end{bmatrix} \quad (13) \\
&\in \mathbb{R}^{Nm \times Np} \\
\mathbf{h} &:= [h_0^T, \dots, h_{N-1}^T]^T \in \mathbb{R}^{Nm}.
\end{aligned}$$

**Remark 4:** The formulation (12) sets the inputs to be affine functions of normally distributed random variables having an unbounded value range. Input constraints can thus not be guaranteed for all possible outcomes of the disturbance, which renders infeasible optimization problems, unless  $M = 0$ . A possible approach to addressing this issue is to relax the hard input constraints and restrict the constraint satisfaction to subsets with prescribed probability levels. We thus define chance constraints also on the inputs, not only on the states, as follows

$$P[\mathbf{S}_j \mathbf{u} \leq \mathbf{s}_j] \geq 1 - \alpha_{u,j}, \quad \forall j = 1, \dots, q. \quad (14)$$

Since the constraints on inputs are hard constraints, it is desirable to impose a higher probability of satisfaction on input constraints. We denote this probability level by  $1 - \alpha_{u,j}$  for constraint  $j$ .

As a second step we do a deterministic reformulation of the chance constraints on the states and on the inputs. With the affine disturbance feedback, the chance constraints on the states are now of the form

$$P[\mathbf{G}_i(\mathbf{A}x_0 + \mathbf{B}\mathbf{M}\mathbf{w} + \mathbf{B}\mathbf{h} + \mathbf{E}\mathbf{w}) \leq \mathbf{g}_i] \geq 1 - \alpha_i. \quad (15)$$

Note that the functions describing the constraints in (15) are bi-affine in the decision variables and the disturbances. It is well-known that if the disturbance is normally distributed, the functions are bi-affine in the decision variables and the disturbances are considered in the constraints, then individual chance constraints can be equivalently formulated as deterministic second order cone constraints [13] as follows

$$\Phi^{-1}(1 - \alpha_i) \|\mathbf{G}_i(\mathbf{B}\mathbf{M} + \mathbf{E})\|_2 \leq \mathbf{g}_i - \mathbf{G}_i(\mathbf{A}x_0 + \mathbf{B}\mathbf{h}) \quad (16)$$

where  $\Phi$  is the standard Gaussian cumulative probability function. The inequalities (16) are second order cone constraints that are convex in the decision variables  $\mathbf{M}$  and  $\mathbf{h}$ .

We obtain the following convex deterministic second-order cone program (SOCP).

**Problem 3:**

$$\begin{aligned}
(\mathbf{M}^*(x_0), \mathbf{h}^*(x_0)) &:= \arg \min_{(\mathbf{M}, \mathbf{h})} \mathbf{c}^T (\mathbf{M}\bar{\mathbf{w}} + \mathbf{h}) \\
s.t. \quad &\Phi^{-1}(1 - \alpha_i) \|\mathbf{G}_i(\mathbf{B}\mathbf{M} + \mathbf{E})\|_2 \leq \mathbf{g}_i - \mathbf{G}_i(\mathbf{A}x_0 + \mathbf{B}\mathbf{h}) \\
&\Phi^{-1}(1 - \alpha_{u,j}) \|\mathbf{S}_j \mathbf{M}\|_2 \leq \mathbf{s}_j - \mathbf{S}_j \mathbf{h}
\end{aligned}$$

**Remark 5:** Please note that the expected value of the linear cost is only affected by the mean, not the covariance.

### C. Performance Bound

PB is not a controller that can be implemented in reality; it is a concept. PB is defined as optimal control with perfect weather and internal gains predictions and thus gives an ultimate bound on what any controller can achieve.

**Remark 6:** In order to compute the PB, we solve an MPC problem, but with perfect weather predictions and a very long prediction horizon (6 days).

## IV. CONTROLLER ASSESSMENT CONCEPT AND SIMULATION SETUP

### A. Controller Assessment Concept

The aim is to estimate the potential of using MPC and weather predictions in building climate control. For this purpose the simulation study was carried out in two steps, which is shown in Figure 2:

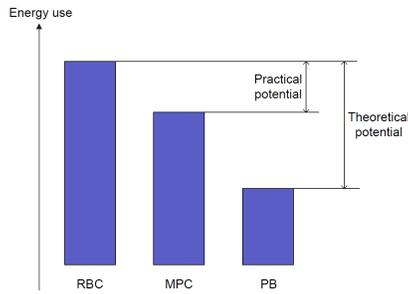


Fig. 2. **Controller assessment concept.** First the theoretical potential is assessed (comparison of RBC and PB), then the practical potential is assessed (comparison of RBC and MPC).

- 1) Theoretical potential: The first step consists of the comparison of RBC and PB. This is done because there is only hope for a significant improvement, if the gap between RBC and PB is large. This investigation is done in a systematic large-scale factorial simulation study for a broad range of cases representing different buildings and different weather conditions as described below. For further details see [7], [8].
- 2) Practical potential: In this investigation we compare the performance of RBC and MPC strategies only for selected cases from the theoretical potential study. Further details can be found in [12].

### B. Simulation Setup

For the potential assessment there were on total 1228 cases considered. The variants were done for the HVAC system, the building itself and its requirements, and the weather conditions. The different variants are listed here:

- HVAC system: Considered are five building system variants (cf. Section II. A).
- Building: The factors vary in building standard (Passive House/Swiss Average), construction type (light/heavy), window area fraction (low/high), internal gains level (occupancy plus appliances; low/high; also associated CO<sub>2</sub>-production), facade orientation (N or S for normal offices, and S+E or S+W for corner offices).

- Weather conditions: We used weather data from four locations (Lugano, Marseille, Zurich, Vienna) being representative for different climatic regions within Europe. All weather predictions and observations were historical data of 2007.

## V. RESULTS

### A. Theoretical Potential Analysis

Evaluated is the annual total (all automated subsystems) non-renewable Primary Energy (NRPE) usage and the annual amount of thermal comfort violations (integral of room temperature above or below comfort range limits). Here we report comparison results for the found 1228 cases where the amounts of violations by RBC are  $< 300$  Kh/a. Figure 3 shows the joint cumulative distribution function of the theoretical energy savings potential (as additional energy use in % of PB) and the amount of comfort violations in Kh/a. It can be seen that more than a half of the considered cases show an additional energy use of more than 40 %. Thus, for many cases there is a significant savings potential, which can potentially be exploited by MPC.

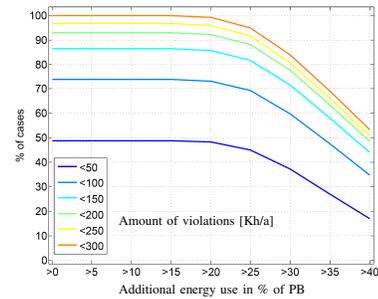


Fig. 3. Joint cumulative distribution function of a particular additional energy use with RBC in % of PB and a particular amount of violations in Kh/a.

### B. Practical Potential Analysis

With real weather predictions, it can happen that constraints are being violated. Therefore, controller performance is assessed in terms of both energy usage and constraint violation.

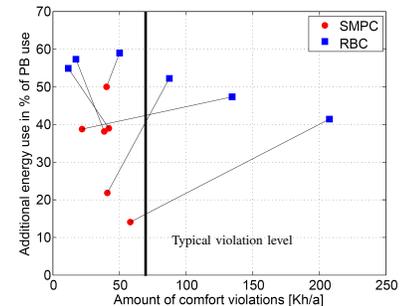


Fig. 4. Comparison of SMPC and RBC.

Figure 4 depicts the result of the comparison of SMPC and RBC for the selected set of experiments: MPC has always clearly less energy use than RBC and in four of six cases smaller amounts of violations. This indicates that the

additional energy use with RBC can be reduced significantly with MPC.

Figures 5 and 6 show the resulting room temperature profile throughout the whole year for one of the selected cases with the RBC and SMPC respectively. It can be seen that SMPC has smaller and less frequent violations than RBC. Furthermore, the diurnal temperature variations are much smaller with SMPC, which is a much more favourable behavior for the room comfort.

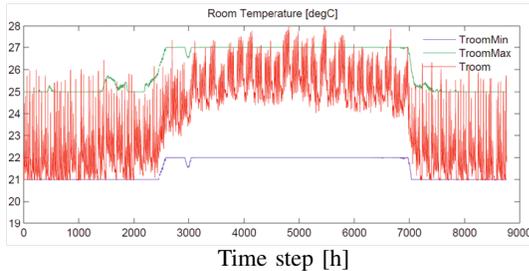


Fig. 5. Room temperature profile of RBC for one year.

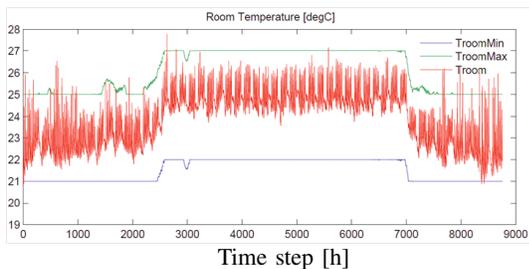


Fig. 6. Room temperature profile of SMPC for one year.

Usage of CE for the six selected cases generally yield much more violations than allowed in the standards (results not shown). One can however tune CE by assuming a tighter comfort band for the controller, which results in less violations and more energy use. Thus, for different comfort band widths one gets a tradeoff curve between energy use and violations. Similarly, SMPC can be tuned. This is however much easier, since there exists a natural tuning knob, the control parameter  $\alpha$ , that describes the probability level of constraint violation.

Figure 7 shows the tuning curves of SMPC and CE for one month (January 2007) as well as the corresponding results of PB and RBC. It can be seen that PB shows no violations and the smallest energy usage as expected. Further can be seen that both CE and SMPC controllers can achieve a better tradeoff between energy use and probability of constraint violations than RBC by moving along the tuning curve. The closer to the origin a tuning curve lies, the better the control performance. Thus, SMPC performs clearly better than both, CE as well as RBC.

## VI. CONCLUSIONS

A Stochastic Model Predictive Control (SMPC) strategy was applied to building climate control. The controller makes use of weather predictions to compute how much energy

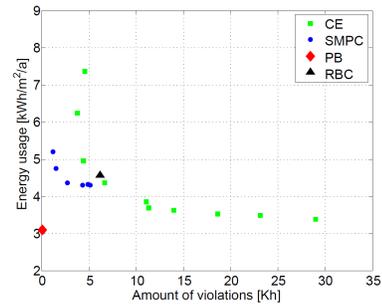


Fig. 7. Tradeoff for SMPC and CE between energy use and violation and comparison with PB and RBC.

and which low/high-cost energy sources are needed to keep the room temperature in the required comfort levels. SMPC was shown to outperform both rule-based control (RBC) as well as a predictive non-stochastic controller (CE). Further benefits of SMPC are easy tunability with a single tuning parameter describing the level of constraint violation as well as comparatively small diurnal temperature variations.

## VII. ACKNOWLEDGMENTS

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